Effect of Lossy Cladding on Modal Dispersion Characteristics of Parabolic-Index Fiber

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***Abstract*—The influence of the cladding on the modal dispersion characteristics of a multimode fiber has been studied for the special case of a lossless fiber.**

In this paper, we consider a multimode fiber with a lossy cladding and explain the influence of the cladding loss on the modal dispersion characteristics of the fiber. We adopt a standard deviation of group delay of guided modes to estimate the modal dispersion behavior of the fiber. We conclude that a lossy cladding improves the modal dispersion characteristics in the case of the parabolic-index fiber.

I. Introduction

Modal dispersion in multimode optical fibers is caused by the differences of group velocities of each guided mode. The modal dispersion broadens the output pulsewidth of the fiber, and in turn restricts the optical data transmission capabilities. It is well known that optical fiber with parabolic-index distribution shows little modal dispersion in comparison with the other optical fibers.

The modal dispersion of the parabolic-index fiber comes mainly from group velocity difference between modes near cutoff frequency and the other well-guided modes. Therefore, there are two methods to obtain a parabolic-index fiber with minimum modal dispersion. One method is to design a fiber with minimum group velocity difference between modes near cutoff frequency and well-guided modes. This is accomplished by truncation of refractive index at core-cladding boundary [1], [2]. The other method is to filter out the modes near cutoff frequency. This will be accomplished by introduction of cladding loss.

In this paper, we study the effect of cladding loss on the modal dispersion characteristics, and show that the modal dispersion characteristics are greatly improved by introduction of cladding loss. The standard deviation of group delays of every guided modes weightened by the power transmission coefficient of the individual guided mode in the fiber gives a good estimate of the modal dispersion characteristics of the fiber, provided that mode conversion effects are small. The power transmission coefficient is easily calculated using the perturbation theory.

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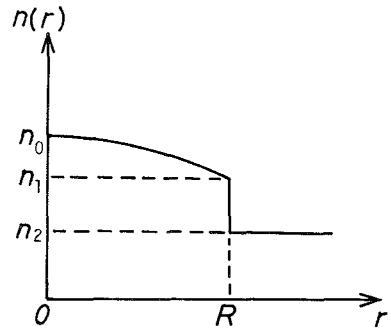

Fig. 1. Refractive index distribution of parabolic-index fiber. R is the radius of the core.

II. Guided Modes of a Parabolic-Index Fiber

The refractive index profile of a parabolic-index fiber is shown in Fig. 1 and given by

$$n^2(r,\theta,z)=\begin{cases} n_0^2-(n_0^2-n_1^2)\cdot(r/R)^2, & r\leqslant R \\ n_2^2, & r>R \end{cases} \tag{1}$$

where n_0, n_1, and n_2 are the refractive indices on the center of the core, on the core side of the core-cladding boundary, and on the cladding, respectively, r, θ, and z are the circular cylinder coordinates, and R is the core radius. An explicit vector analysis of a parabolic-index fiber is complicated, so we employ the usual scalar analysis. A field $\Phi(r,\theta,z)$ satisfies

$$\nabla^2\Phi+k^2n^2\Phi=0 \tag{2}$$

where k is a wavenumber in free space. Assuming that the direction of propagation is along the z axis, the field Φ can be expressed as

$$\Phi(r,\theta,z)=\phi(r,\theta)\cdot\exp(-j\beta z) \tag{3}$$

where β is the propagation constant. Substituting (3) into (2), we get

$$\nabla^2\phi+(k^2n^2-\beta^2)\phi=0. \tag{4}$$

The scalar wave equation (4) can be solved exactly when the refractive index n is given by (1). The field in the core which is regular on $r=0$ is given by

$$\phi_{\text{core}}(r,\theta)=A\cdot L_\mu^l(u[r])\cdot u(r)^{1/2}\cdot\exp(-u[r]/2)\cdot\begin{bmatrix}\sin(l\theta)\\ \cos(l\theta)\end{bmatrix} \tag{5}$$

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where A is an arbitrary constant, L_μ^l is a Laguerre function, l is zero or a positive integer, and μ, which is not an integer in this case, and $u(r)$ are expressed as

$$\mu = \frac{R(k^2n_0^2-\beta^2)}{4k\sqrt{n_0^2-n_1^2}} - \frac{l+1}{2} \tag{6}$$

and

$$u(r) = kR\sqrt{n_0^2-n_1^2}\cdot(r/R)^2 \tag{7}$$

respectively. The field in the cladding which converges to zero at $r=\infty$ is expressed as

$$\phi_{\text{clad}}(r,\theta) = B\cdot K_l(v[r])\cdot\begin{bmatrix}\sin(l\theta)\\ \cos(l\theta)\end{bmatrix} \tag{8}$$

where B is an arbitrary constant, K_l is a modified Bessel function of the second kind, and $v(r)$ is given by

$$v(r) = \sqrt{\beta^2 - k^2n^2\cdot r}\ . \tag{9}$$

Equations (5) and (8) are specified by the following boundary condition:

$$\phi_{\text{core}}\big|_{r=R} = \phi_{\text{clad}}\big|_{r=R} \tag{10}$$

$$\left.\frac{\partial\phi_{\text{core}}}{\partial r}\right|_{r=R} = \left.\frac{\partial\phi_{\text{clad}}}{\partial r}\right|_{r=R}. \tag{11}$$

Substituting (5) and (8) into (10) and (11), we obtain two linear relations between A and B. Eliminating A and B from this pair of relations, we get a characteristic equation which specifies the possible value of β for guided modes:

$$2u(R)\cdot\left[1-\left(\frac{\mu}{l+1}+1\right)\cdot\frac{L_\mu^{l+1}(u[R])}{L_\mu^l(u[R])}\right] = -\frac{v(R)}{2K_l(v[R])}\cdot\left[K_{l-1}(v[R])+K_{l+1}(v[R])\right] \tag{12}$$

and

$$kn_2 < \beta < kn_0. \tag{13}$$

In (12), the $(m+1)$th eigenvalue β counted from the maximum one is the propagation constant β_{lm} of the (l,m) mode, where m is zero or a positive integer. We can obtain the group delay and the power distribution of the (l,m) mode by using the propagation constant β_{lm}.

So far we have considered the lossless case. We now consider that a little loss is introduced uniformly into the cladding. With the bulk attenuation factor a_{clad} of the cladding material, the attenuation factor of the (l,m) mode is given by

$$a_{lm} = \kappa_{lm}\cdot a_{\text{clad}} \tag{14}$$

where κ_{lm} is a proportion of the power of the (l,m) mode which propagates in the cladding and can be easily calculated from the power distribution of the (l,m) mode. While the influence of the cladding on the well-guided lower order modes is negligible, the higher order modes near cutoff are affected significantly by the cladding. The higher order modes have large κ_{lm} and are expected to be strongly attenuated by introduction of cladding loss, and, consequently, the high attenuation of these modes will decrease the modal dispersion of the fiber. Assuming that all guided modes are uniformly excited at the sending end of the fiber and that mode conversion effects are negligible, the average loss of all guided modes per length L is given by

$$L_{\text{av}} = \frac{1}{N}\cdot\sum_{lm}\exp(-a_{lm}\cdot L) \tag{15}$$

where N is the total number of guided modes, and Σ denotes the sum over all guided modes. In addition, the bulk loss of the cladding material per length L is shown by

$$L_{\text{clad}} = \exp(-a_{\text{clad}}\cdot L). \tag{16}$$

The delay time τ_{lm} of the pulse of the (l,m) mode after propagation along the fiber of length L is given by

$$\tau_{lm} = \frac{L}{c}\cdot\frac{d\beta_{lm}}{dk} \tag{17}$$

where c is the velocity of light in free space. Because the delay times τ_{lm} are different from each other, the input pulsewidth is broadened after propagation along the multimode fiber. To estimate the broadening of the output pulsewidth, we introduce the standard deviation S of the group delay $d\beta_{lm}/dk$ of all guided modes defined by

$$S = \left[\frac{1}{M}\cdot\sum_{lm}\exp(-a_{lm}\cdot L)\cdot\left[\frac{d\beta_{lm}}{dk} - \left\{\frac{1}{M}\sum_{l'm'}\exp(-a_{l'm'}\cdot L)\cdot\frac{d\beta_{l'm'}}{dk}\right\}\right]^2\right]^{1/2} \tag{18}$$

where

$$M = \sum_{lm}\exp(-a_{lm}\cdot L). \tag{19}$$

In (18), we used the power transmission coefficient exp $(-a_{lm}\cdot L)$ of each guided mode as a weighting factor because of the assumption that all guided modes are uniformly excited and that the mode conversion effects are negligible all over the fiber. The standard deviation S is related directly to the output pulse shape. The width $\Delta\tau$ over which the magnitude of the output pulse shape decays by a factor of $1/e$ is given approximately by

$$\Delta\tau = 3.3\frac{L}{c}\cdot S. \tag{20}$$

The above relation shows that the broadening of the output pulsewidth is proportional to the standard deviation S.

III. Numerical Examples and Discussion

Let us introduce the normalized frequency

$$\omega_n = kR\sqrt{n_1^2-n_2^2} \tag{21}$$

to treat uniformly the modal dispersion characteristics of

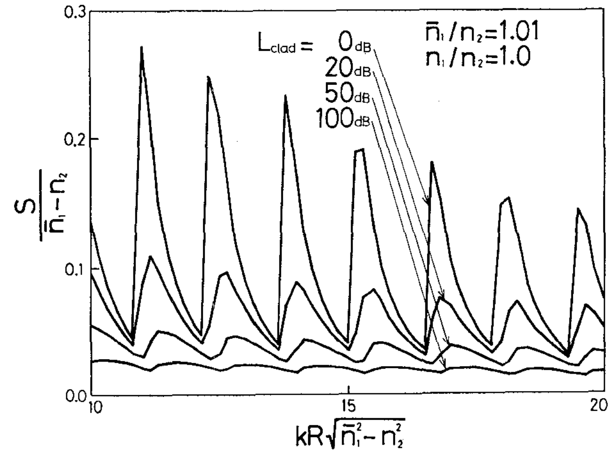

Fig. 2. Normalized modal dispersion $S/(\bar{n}_1-n_2)$ versus normalized frequency $\omega_n=kR\sqrt{\bar{n}_1^2-n_2^2}$ with L_{clad} as a parameter. $\bar{n}_1/n_2=1.01$, $n_1/n_2=1.0$.

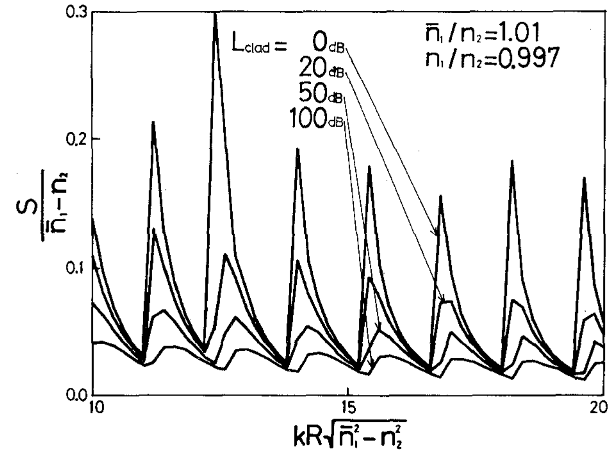

Fig. 4. Normalized modal dispersion $S/(\bar{n}_1-n_2)$ versus normalized frequency $\omega_n=kR\sqrt{\bar{n}_1^2-n_2^2}$ with L_{clad} as a parameter. $\bar{n}_1/n_2=1.01$, $n_1/n_2=0.997$.

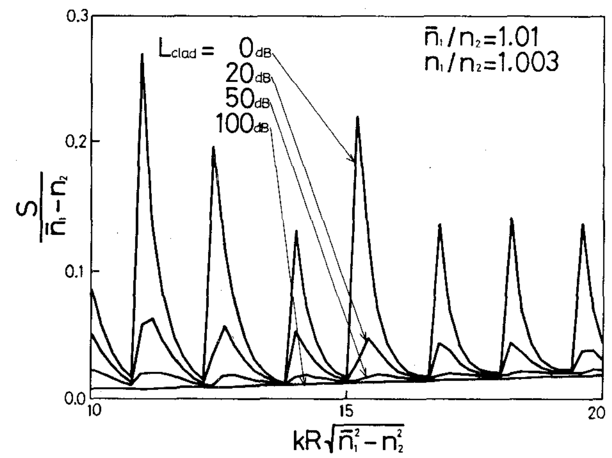

Fig. 3. Normalized modal dispersion $S/(\bar{n}_1-n_2)$ versus normalized frequency $\omega_n=kR\sqrt{\bar{n}_1^2-n_2^2}$ with L_{clad} as a parameter. $\bar{n}_1/n_2=1.01$, $n_1/n_2=1.001$.

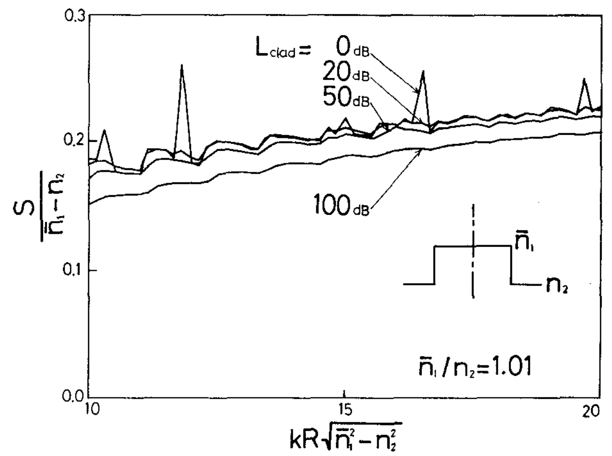

Fig. 5. Normalized modal dispersion $S/(\bar{n}_1^2-n_2^2)$ versus normalized frequency $\omega_n=kR\sqrt{\bar{n}_1^2-n_2^2}$ in the case of the step-index fiber with L_{clad} as a parameter. $\bar{n}_1/n_2=1.01$.

various parabolic-index fibers and the step-index fiber, where $\bar{n}_1$ is an equivalent refractive index of the core defined by

$$\bar{n}_1^2=n_2^2+\frac{2}{R^2}\int\{n^2(r)-n_2^2\}\,rdr. \tag{22}$$

The integral in the above equation must be carried out over the region $n^2(r)>n_2^2$ of the core. The total number N of guided modes in a fiber is closely related to (21) and given by

$$N=\frac{\omega_n^2}{2}. \tag{23}$$

Figs. 2–4 show numerical examples of the frequency characteristics of the normalized standard deviation $S/(\bar{n}_1-n_2)$ for the parabolic-index fibers, where $\bar{n}_1/n_2$ is equal to 1.01, n_1/n_2 is 1.0, 1.003, and 0.997, respectively, and L_{clad} is assumed to be 0 dB, 20 dB, 50 dB, and 100 dB, respectively. As we can see from these figures, the normalized standard deviation $S/(\bar{n}_1-n_2)$ varies abruptly and almost periodically in the lossless case ($L_{clad}=0$ dB). This feature is characteristic for the parabolic-index fiber and caused by the fact that the guided modes with identical value of $(2m+1)$ have almost the same propagation constant, and, therefore, the new guided modes begin to propagate in a group with increasing normalized frequency ω_n. Furthermore, Figs. 2–4 show that introduction of cladding loss reduces the peak values of ripples on the normalized standard deviation $S/(\bar{n}_1-n_2)$ curves and restrains the influence of the group of modes near cutoff on the modal dispersion. These effects become larger with increasing cladding loss and increasing value of n_1/n_2. The dependence of these effects on the value of n_1/n_2 is due to the fact that the rate of change of the refractive index in the core becomes larger for smaller value of n_1/n_2, and that consequently the guided mode power of the fiber with smaller n_1/n_2 is trapped more tightly in the core than with larger n_1/n_2.

For cladding losses L_{clad} higher than 100 dB, the guided modes near cutoff attenuate strongly and the ripples on the normalized standard deviation $S/(\bar{n}_1-n_2)$ curves almost vanish. Therefore, a cladding loss $L_{clad}=100$ dB is

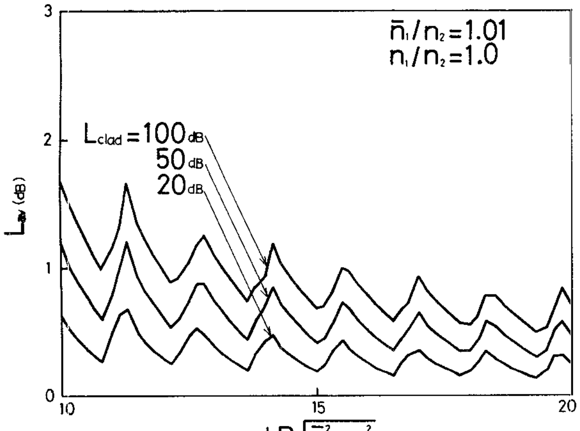

Fig. 6. Averaged transmission loss of the parabolic-index fiber L_{av} versus normalized frequency $\omega_n = kR\sqrt{\bar{n}_1^2 - n_2^2}$ with L_{clad} as a parameter. $\bar{n}_1/n_2 = 1.01$, $n_1/n_2 = 1.0$.

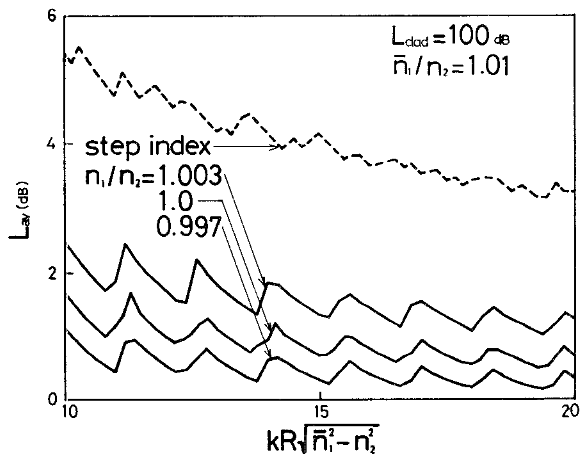

Fig. 7. Averaged transmission loss L_{av} versus normalized frequency $\omega_n = kR\sqrt{\bar{n}_1^2 - n_2^2}$ in the case of parabolic-index fiber for $n_1/n_2 =$ 1.003, 1.0, and 0.997 and in the case of step-index fiber. $L_{clad} = 100$ dB, $\bar{n}/n_2 = 1.01$.

sufficient for the purpose of improving the modal dispersion.

Fig. 5 shows the frequency characteristics of the normalized standard deviation $S/(\bar{n}_1 - n_2)$ in the case of the step-index fiber, where $\bar{n}_1/n_2$ is equal to 1.01 and the cladding losses are chosen to be 0 dB, 20 dB, 50 dB, and 100 dB. As shown in this figure, cladding loss has almost no effect on the improvement of the modal dispersion characteristics of the step-index fiber.

The frequency characteristics of the averaged transmission loss of the parabolic-index fiber are shown in Fig. 6, where values of n_1/n_2 and $\bar{n}_1/n_2$ are assumed to be 1.0 and 1.01, respectively. From this figure it is apparent that the averaged transmission loss L_{av} is at most 1.0 dB, even if the cladding loss L_{clad} is 100 dB.

Fig. 7 shows the frequency characteristics of the averaged transmission loss L_{av} in the case of the parabolic-index fiber for $n_1/n_2 = 1.003$, 1.0, and 0.997, and in the case of the step-index fiber, where we assumed L_{clad} to be 100 dB and $\bar{n}_1/n_2$ to be 1.01. As we can see from this figure, the averaged transmission loss L_{av} of the step-index fiber is much larger than that of the parabolic-index fibers, and L_{av} of the parabolic-index fiber becomes larger with increasing value of n_1/n_2. As a result, it is clear that cladding loss does not improve the modal dispersion characteristics of step-index fibers and that the method of obtaining multimode fibers with minimum modal dispersion by introduction of cladding loss is better suited to parabolic-index fibers.

IV. Conclusion

We have considered a method of obtaining parabolic-index fibers with low modal dispersion by introduction of cladding loss. As a result, it has been explained that introduction of cladding loss improves the modal dispersion characteristics of the parabolic-index fiber without a significant increase of the transmission loss, and consequently that this method is effective. On the contrary, the introduction of cladding loss in the case of step-index fiber is not effective.

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